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Young Children Exploring Early Calculation

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In this paper we outline one part of our extensive study into children's own mathematical graphics.

Counting has been identified by many as being one of the significant ways in which young children develop mathematical understanding. This has been well documented [1,2]. Gelman and Gallistell found that many young children under the age of three had already developed useful knowledge of counting [I]. Gelman went on to claim that children are born with an innate learning devise, a non-verbal counting mechanism, called an accumulator [2]. From this basic understanding of counting children go on to use this knowledge for more complex number operations.

To support this development of numeracy much of the most recent literature on teaching has concentrated on supporting counting strategies [3]. This has been well received by teachers and many practical teaching ideas have ensued. However what has proved more difficult and less clearly defined is when and how to teach standard symbols and when to introduce the standard calculation eg, '7 + 3 = 10'. Gifford states that teachers can teach children to do these algorithms but they may have little understanding of how to apply this because they memorise a trick and the calculation is not related to what they know and can do (4).

The national numeracy strategy in England recommends that children's own methods of calculation are supported as they develop understanding [3]. This is also supported by the Foundation Stage Guidance, 'asking children to "put something on paper" about what they have done or found out will allow them to choose how to record' [5]. The QCA publication Teaching written calculations [6] provides some examples of possible ways children might choose to put down their thoughts on paper but until now there has been little research on children's own approaches. There is also an important distinction between the children recording something after they have done some practical mathematics and thinking about the mathematics as they are doing it. The paper and pencil in the latter become tools for sketching ideas not yet done. This is vitally important for encouraging children's understanding of calculations. In our own study [7] of 273 teachers of children in the 3 to 8 age range we found that less than 10 per cent gave children opportunities to make their own mathematical marks, Yet we know that children have great difficulties with understanding algorithms [4,8]. Many teachers say that they use practical maths instead of giving children any opportunities to write down their mathematical thinking [10]. Yet as Askew states there must be an element of 'in the head maths' because children might think maths is always practical. He goes on to say that mental mathematics provides the images that they may explore on paper, [9].

Not only is mental mathematics vital for children's representations, but the way in which mental mathematics is taught is similar to the teaching we advocate for supporting children's own mathematical graphics and written methods. It is from this pedagogy that we have uncovered children's mathematical meaning on paper.

In our research we analysed over 700 examples of children's own mathematics on paper.

We categorised these into:

- Forms -these are types of marks children make when exploring any of the dimensions explained below. These forms of mathematical graphics are similar to Hughes types [8].
- Dimensions -there are five dimensions of mathematical graphics. This is the mathematics children are using when they are putting their mathematical thinking on paper.

There are five dimensions:

1 Early play with objects
2 Early written numerals
3 Numerals as labels
4 Representations of quantities and counting
5 Early operations: the development of children's own written methods.

In this article we are going to focus on the fifth dimension, *early operations: the development of children's own written methods*. Within this dimension children start exploring symbols but many use *counting continuously*, for addition (dimension 5a):

Dimension 5: Early operations: the development of children's own written methods:
(a) counting continuously
(b) separating sets
(c) exploring symbols

This is where they count the first set as they make their marks and continue to count the second set putting the marks beside the first. Many of the children in the study then counted all the marks, whether it be tallies or pictures, for the final total. For subtraction they again count the first set and then in different ways 'take away' the second set.

The 'melting pot'

As children move beyond counting continuously they move into a phase of thinking about operations more and we saw that children start to separate the sets (5b) and explore symbols (5c). At this stage we saw a wide variety of different representations which we refer to as the 'melting pot'. Carpenter and Moser [10] studied the range of strategies children use to add and subtract and found that a variety of informal counting persists through the primary years.

Children's choices and ways to represent do not remain static since discussion modelling and conferencing alters individuals' perspectives and continuously introduces them to further possibilities. The melting pot period therefore covers a huge range of representations through an extended period of time. Some children in this period may choose different ways of representing their calculations as they refine their ways and ideas of representing addition and subtraction.

Separating sets

Children use a range of strategies to show that the two amounts are distinctly separate. They do this in a variety of ways including:

- grouping the two sets of items to be added either on opposite sides (to the left and right) of their paper, or by leaving a space between them
- separating the sets with words
- putting a vertical line between sets
- putting an arrow or a personal symbol between the sets

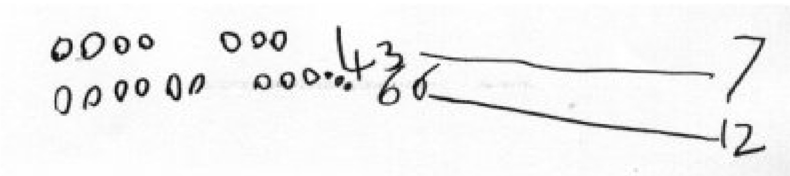

Figure 1: Jack – 5 years 3 months

Transition periods

Jack (Figure 1) has used at first a space between the sets but he has gone on to use standard symbols and a long single line for the equals sign. We would say that Jack is in a transition period where he uses combinations of standard signs and his own invented symbols. He also began with separating sets, a procedure he was comfortable with. Jack then decided to further communicate using more standard methods. This transition period is vital to his self-awareness about moving into standard symbols. He is starting to make connections between his own invented methods that he under- stands -to the abstract symbolism of mathematics which he does not understand so well. Jack will soon be discarding the separating sets method to use symbols and signs as a quicker way to do calculations. We emphasise again the importance of giving children opportunities to understand calculations in their own way. If they are allowed to use their own methods they will move securely into standard calculations.

Using implicit symbols

Through the marks they make or the layout of their calculation, children often show that the symbol is implied and that they understand the calculation in their head. We believe this represents a highly significant point in children's developing under- standing. Jax (Figure 2) had been adding two small quantities of grapes. He implied the '+ ' symbol by leaving a space between the dots and read this as '6 and 4': pointing to the 'T' he read 'ten' (the total).

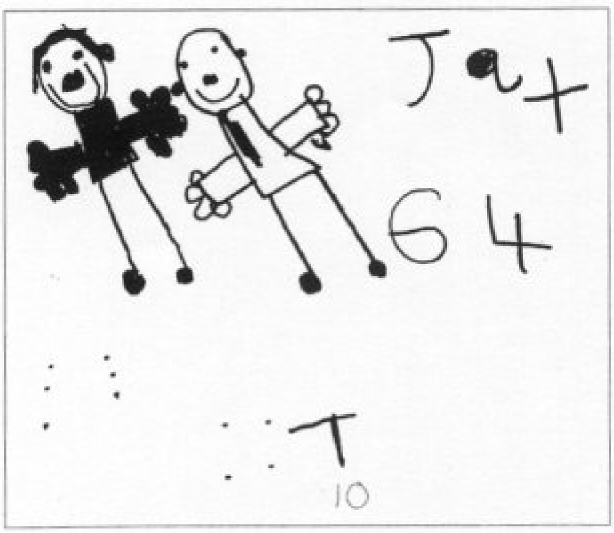

Figure 2: Jax – 5 years 2 months

Code switching

Code switching occurs when a speaker switches from one language to another in mid-sentence; for example, *'J'ai mangé du fish and chips aujourd'hui'*. Examples of code-switching within the writing of young bi-lingual children have also been found. Our research has highlighted children's use of code- switching within their early 'written' mathematics [7]. We found that the most significant switches occur when children move between:

- implicit symbols
- their own symbols
- standard symbols

Barney [Figure 3] began by using a combination of number and letters writing' 10 take 1 is 9' .

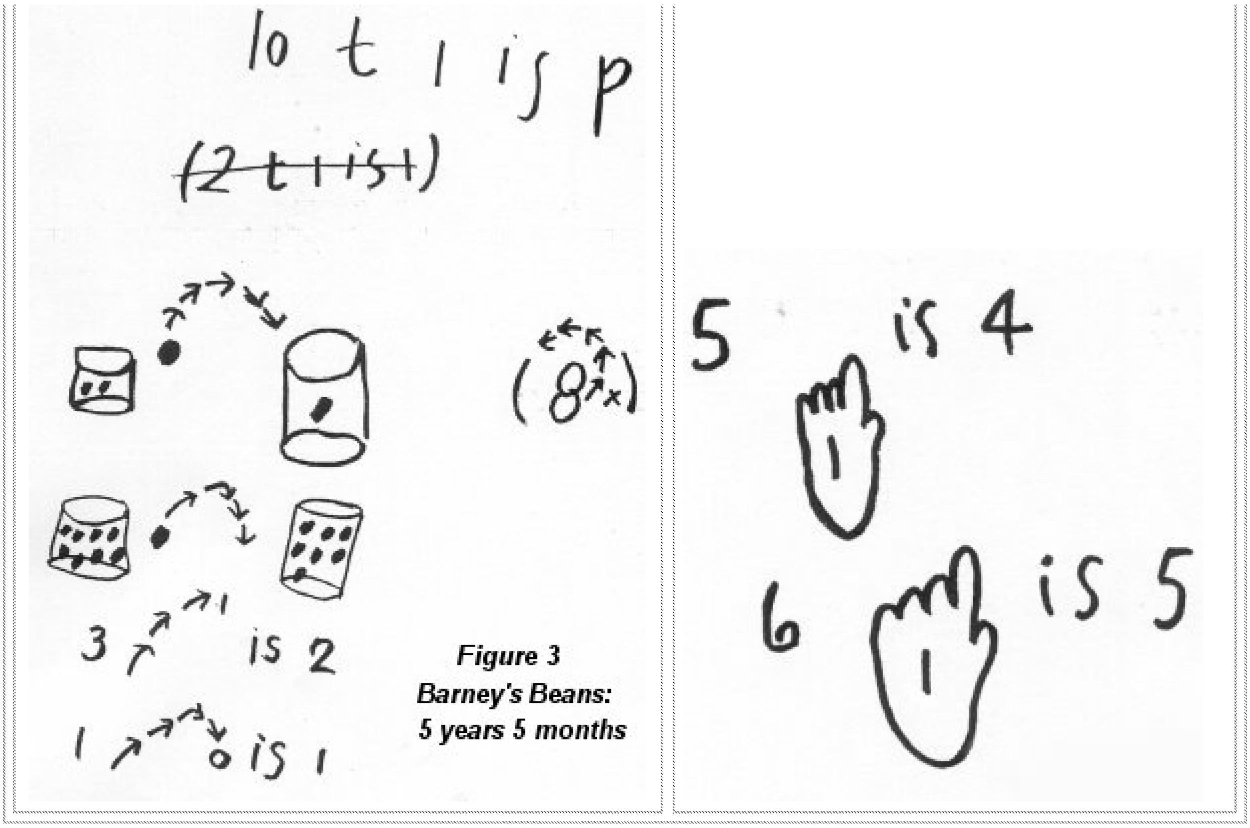

Figure 3
Barney's Beans:
5 years 5 months

Through successive shorthand he experimented with arrows to show the action of removing one bean in his game with beans and flowerpots. At the foot of the page he used numerals rather than beans and flowerpots. Barney was the first child in the class to use arrows to denote 'take away'. Finally he substituted a hand with a number '1' written on it to show that his hand had removed one bean. In several of the calculations Barney used the word 'is' to stand for the symbol '='.

To ensure that all the features of the narrative are understood, children use talk or sometimes add writing to help adults understand the meanings of their graphics. In figure 4 Louisa has combined pictorial and written forms with the symbol '6'. She used words as she experimented with the role and function of symbols. She read her calculation as 'I had two strawberries and then I got four more. Then there's six altogether'.

Figure 4: Louisa - 5 years 1 month

The invented symbols children add may show something they have done in a concrete situation. We term children's use of hands and arrows narrative action. In figure 5, Fred separated two sets of grapes he was adding with a line. The plus and equals symbols are implied and can be read as '5 plus I equals 6'. He drew his hand adding one
grape and finally write the number '6' (the total) beneath. He had begun this calculation by attempting us use a standard symbol (' 5 = I') but discarded this to use forms of representation that built on his understanding. Exploring symbols in their own ways leads children to use to standard symbolic calculations with small numbers.

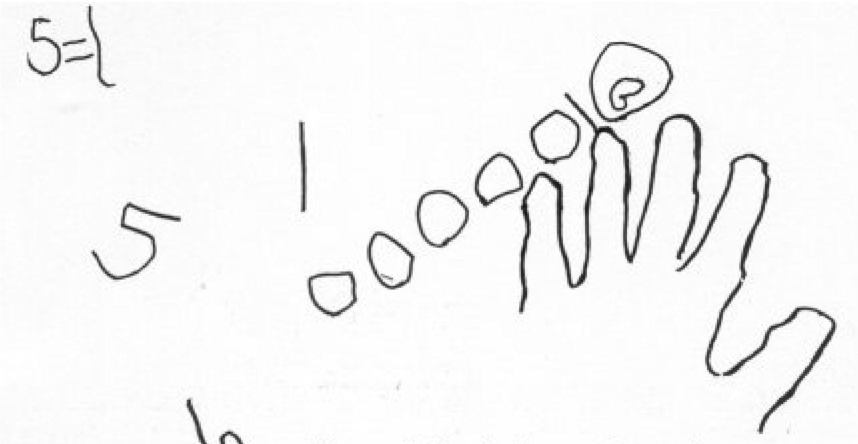

Figure 5: Fred - 5 years 8 months

Why encourage children to use their own ideas in mathematics?

Working in this way allows children to become what we term 'Bi-numerate' -able to translate between their informal, home mathematics and the abstract written mathematics of school. They come to understand the standard mathematical symbols and written algorithms as a deep level. Recognition of the difficulties young children experience with standard symbols and calculations led to recognition of the need to build on their personal mental methods (4, 8,11). Unfortunately this understanding has until now been largely unconnected to children's written mathematics in the classroom.

This is the first time children's own calculations from the age of 3 - 8 years have been analysed in detail, using an extensive range of examples from real teaching contexts (7). We have shown how a child's growing understanding of the role of symbols and the need to communicate their personal mathematical thinking contributes to their understanding of the mathematics.

Young mathematicians

What does it mean to encourage children to 'see themselves as mathematicians' [5]? Further support for a bi-numerate approach comes from Holton et al [12]. They propose that 'in the context of written calculations'. ..'mathematical play' with ideas and processes 'seems a natural strategy, undertaken by young children and research mathematicians alike', (p.413). Thompson supports this way of working, emphasising that teachers must ensure that 'substantial emphasis (is) given to mental methods *and to the development of children's own methods of recording at Key Stage l* ' (p.l06, 13). However, in practice this link is seldom made by children in Foundation and Key Stage 1.

We argue that to become young mathematicians 'requires creative thinking, an element of risk-taking, imagination and invention - dispositions that are impossible to develop within the confines of a work-sheet or teacher-led written mathematics' (p,23: 14), It is interesting to see the extent to which teachers now use 'open' questions in mental/oral mathematics, but persist in providing 'closed' pieces of written work. If we wish children to develop understanding of standard mathematical symbols and calculations as a deep level, we need to value and support children 'playing' with mathematical ideas and exploring their mathematical thinking on paper in open ways.

It is important to emphasise that none of these features were taught and that this range often originates from children's personal ways of thinking. Implicit and invented symbols, code-switching and narrative action are powerful indicators of mathematical thinking and meaning making. When children's early calculations show that they are moving between two dimensions - such as Jack, Barney and Fred - teachers can be confident that this is evidence of progression towards a secure understanding of abstract symbolism.

The different dimensions we have discussed here are essential stages in children's mathematical development. Significantly they pave the way for the next stage in which children freely choose to use standard symbols, and do so with genuine understanding.

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